1A.

510 = 001012

-1010 in signed 5-bit binary: 1010 = 010102, invert it then +12 🡪 -1010 = 101012 + 12 = 101102

1B.

5 \* -10 = -50, or 5 \* 10 = 50, then 50 \* -1 = -50.

5 = 00101, 10 = 01010.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 01010  00101  01010 | 01010  00010|1  00000|0 | 01010  00001|01  01010|00 | 01010  00000|101  00000|000 | 01010  00000|0101  00000|0000 | 01010  00000|00101  00000|00000 |

|  |
| --- |
| 0000010000 |
| 01010  000000  0101000  00000000  000000000  0000000000 |
| 0000110010 |

Find two’s compliment: invert the result: 00001100102 🡪 11110011012, + 12 = 11110011102.

1C-1. Zero extension from 5-bit to 8-bit:

510 = 001012. Add zeros in the front of MSB until it is 8-bit: 001012 🡪 000001012.

-1010 = 101102. Add zeros in the front of MSB until it is 8-bit: 101102 🡪 000101102.

1C-2. Signed extension from 5-bit to 8-bit:

510 = 001012. Number is positive, add zeros in the front of MSB until it is 8-bit:

001012 🡪 000001012.

-1010 = 101102. Number is negative, add ones in the front of MSB until it is 8-bit:

101102 🡪 111101102.

2A.

F(x, y, z) = x + y + z'

|  |  |  |  |
| --- | --- | --- | --- |
| X | Y | Z | F(x, y, z) |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

2B.

F(x, y, z) = x'y' + yz

|  |  |  |  |
| --- | --- | --- | --- |
| X | Y | Z | F(x, y, z) |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

3A.

x’z + y + xy’ = x + y + z

x’z + (x + y)(y + y’) = x + y + z

x’z + (x + y)(1) = x + y + z

x’z + (x + y) = x + y + z

x’z + x + y = x + y + z

(x + x’)(x + z) + y = x + y + z

(1)(x + z) + y = x + y + z

x + y + z = x + y + z

3B.

abc’ + bc’d’ + bc + c’d = b + c’d

abc’ + (bc’d’ + c’d) + bc = b + c’d

abc’ + (c’(bd’ + d)) + bc = b + c’d

abc’ + (c’((b + d)(d’ + d))) + bc = b + c’d

abc’ + (c’((b + d)(1))) + bc = b + c’d

abc’ + (c’(b + d)) + bc = b + c’d

abc’ + bc’ + c’d + bc = b + c’d

b(ac’ + c’ + c) + c’d = b + c’d

b(ac’ + (c’ + c)) + c’d = b + c’d

b(ac’ + (1)) + c’d = b + c’d

b(1) + c’d = b + c’d

b + c’d = b + c’d

4A.

f (A, B, C, D) = ∑m (0, 1, 5, 7, 8, 10, 14, 15)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | cd |  |  |  |  |
| ab |  | 00 | 01 | 11 | 10 |
|  | 00 | 1 | 1 |  |  |
|  | 01 |  | 1 | 1 |  |
|  | 11 |  |  | 1 | 1 |
|  | 10 | 1 |  |  | 1 |

0000 0001 0111 1110

1000 0101 1111 1010

B’C’D’ + A’C’D + BCD + ACD’ = f (A, B, C, D)

Prime implicants: **B’C’D’**, **A’C’D**, **BCD**, **ACD’**, all groups listed are in their maximum groupings. Essential prime implicants: There are no essential prime implicants, because there are no 1s in the vertical groupings not covered by the horizontal groupings.

4B.

f (W, X, Y, Z) = ∑m (2, 4, 9, 12, 15) + d (3, 5, 6, 13)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | YZ |  |  |  |  |
| WX |  | 00 | 01 | 11 | 10 |
|  | 00 |  |  | X | 1 |
|  | 01 | 1 | X |  | X |
|  | 11 | 1 | X | 1 |  |
|  | 10 |  | 1 |  |  |

0100 1101 1101 0010

1100 1001 1111 0011

0101

1101

XY’ + WY’Z + WXZ + W’X’Y = f (W, X, Y, Z)

Prime implicants: **XY’, WY’Z, WXZ, W’X’Y**, because those are the maximum groupings available. A group of 4 followed by 3 groups of 2 are the maximum groupings available with adding addition terms (the red groups) to the answer.

Essential prime implicants: **XY’, WY’Z, WXZ, W’X’Y** are all essential prime implicants. 1100 in **XY’** expressed in any other groupings without decreasing the grouping’s size, so it is an essential prime implicant. 1001 in **WY’Z** cannot be grouped by any other way, so that is an EPI. 1111 in **WXZ** cannot be grouped in any other way, it is also an EPI. 0010 in **W’X’Y’** can be grouped with 0110 to form another group. You can select either group to be the essential prime implicant, but doing so will make the other group non-essential, so I chose to group it with 0011 to make **W’X’Y’** an EPI. Additionally, 0110 and 0011 are don’t cares, so if you choose to group with either of them, you can treat the non-grouped one as 0, eliminating the need to group it.

5. Take a 4-bit number, return true if number is between (inclusive) 5 and 12, and if it’s even.

Truth table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Decimal | W | X | Y | Z | f(W, X, Y, Z) |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 |
| 6 | 0 | 1 | 1 | 0 | 1 |
| 7 | 0 | 1 | 1 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 1 |
| 9 | 1 | 0 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 |
| 13 | 1 | 1 | 0 | 1 | 0 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 |

K-Map:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | YZ |  |  |  |  |
| WX |  | 00 | 01 | 11 | 10 |
|  | 00 |  |  |  |  |
|  | 01 |  |  |  | 1 |
|  | 11 | 1 |  |  |  |
|  | 10 | 1 |  |  | 1 |

1100 0110 1000

1000 1010

WY’Z’+W’XYZ’+WX’Z’ = f(W, X, Y, Z)

Schematic diagram:

